

Close Thurs: HW_4A,4B,4C (6.4,6.5)

Close next Wed: HW_5A, 5B, 5C (7.1,7.2,7.3)

6.5 Average Value

The average y -value of $y = f(x)$ from $x = a$ to $x = b$ is given by

$$f_{ave} = \frac{1}{b - a} \int_a^b f(x) dx$$

Entry Task:

The formula for the temperature of a particular object is $T(t) = t^2$ degrees Fahrenheit where t is in hours.

Find the average temperature from $t = 1$ to $t = 4$ hours.

The mean value theorem for integrals:

If $f(x)$ is continuous on from $x = a$ to $x = b$, then there is at least one value $x = c$ at which

$$f(c) = f_{ave}.$$

Example:

Using $T(t) = t^2$ from $t = 1$ to $t = 4$ again.

Find a time at which the temperature is exactly equal to the average value.

Average Value Derivation

The average value of the n numbers:

$$y_1, y_2, y_3, \dots, y_n$$

is given by

$$\frac{y_1 + y_2 + y_3 + \dots + y_n}{n} = y_1 \frac{1}{n} + \dots + y_n \frac{1}{n}.$$

Goal: We want the average value of **all** the y -values of some function $y = f(x)$ over an interval $x = a$ to $x = b$.

Derivation:

1. Break into n equal subdivisions

$$\Delta x = \frac{b-a}{n}, \text{ which means } \frac{\Delta x}{b-a} = \frac{1}{n}$$

2. Compute y -value at each tick mark

$$y_1 = f(x_1), y_2 = f(x_2), \dots, y_n = f(x_n)$$

3. Ave $\approx f(x_1) \frac{\Delta x}{b-a} + \dots + f(x_n) \frac{\Delta x}{b-a}$

$$\text{Average} \approx \frac{1}{b-a} \sum_{i=1}^n f(x_i) \Delta x$$

Thus, we can define

$$\text{Average} = \frac{1}{b-a} \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

Which means the exact average y -value of $y = f(x)$ over $x = a$ to $x = b$ is

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$$

7.1 Integration by Parts

Goal: We will reverse the product rule.

Before we start, add these to your basic list of integrals:

$$\int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + C$$

$$\int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + C$$

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$$

$$\int \frac{1}{ax + b} dx = \frac{1}{a} \ln |ax + b| + C$$

Derivation of Integration By Parts

The product rule says:

$$u(x)v'(x) + v(x)u'(x) = \frac{d}{dx}(u(x)v(x))$$

which can be written as

$$\int u(x)v'(x)dx + \int v(x)u'(x)dx = u(x)v(x)$$

Writing this in terms of the differentials:

$$dv = v'(x)dx \quad \text{and} \quad du = u'(x)dx$$

we have

$$\int u \, dv + \int v \, du = uv$$

which we rearrange to get

Integration by Parts formula:

$$\int u \, dv = uv - \int v \, du$$

Example:

$$\int x \cos(8x) dx$$

Step 1: Choose u and dv .

Step 2: Compute du and v .

Step 3: Use formula (and hope)

Example:

$$\int x^2 \ln(x) dx$$

Example:

$$\int_1^e x^2 \ln(x) dx$$

Notes:

1. The symbols u and v **never** appear in the integration. They are just locations in the formula (no variables are changing, this is not substitution).
2. u and dv completely split up the integrand. **Once you chose u , then dv is everything else.**
3. The goal is to make $\int v du$ “nicer” than $\int u dv$
 - (a) Pick u = “something that gives a derivative that is simpler than the original u ”
 - (b) Pick dv = “something that you can integrate”
 - (c) And hope “ vdu ” is something in our table!

Example:

$$\int x^2 e^{x/2} dx$$

Example:

$$\int e^x \cos(x) dx$$

Example:

$$\int \sin^{-1}(x) dx$$